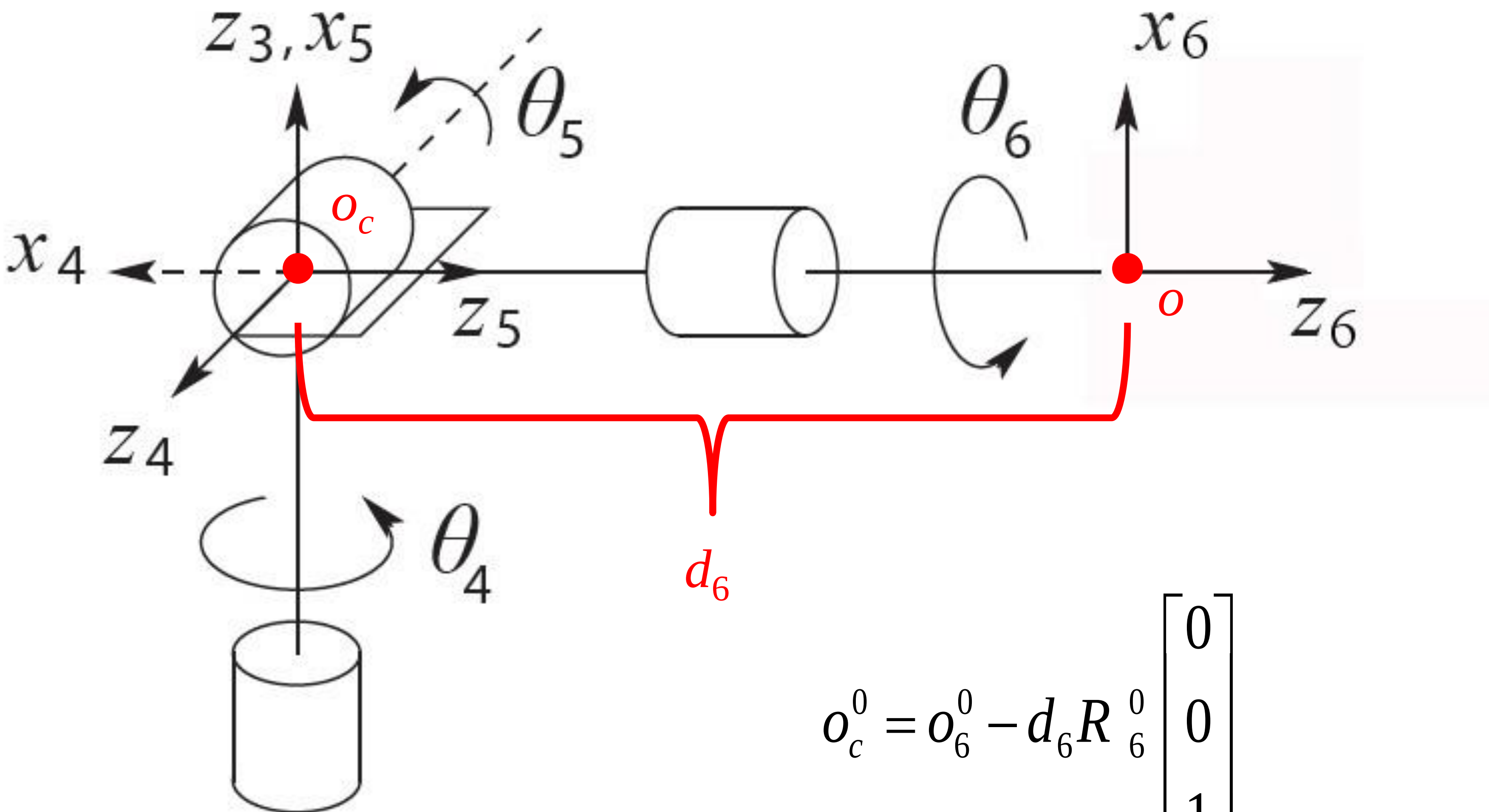
Day 9

Inverse Kinematics; Trajectory Generation

Spherical Wrist



$$o_c^0 = o_6^0 - d_6 R_6^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Inverse Kinematics Recap

1. Solve for the first 3 joint variables q_1, q_2, q_3 such that the wrist center o_c has coordinates

$$o_c^0 = o_6^0 - d_6 R_6^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2. Using the results from Step 1, compute R_3^0
3. Solve for the wrist joint variables q_4, q_5, q_6 corresponding to the rotation matrix

$$R_6^3 = \left(R_3^0\right)^T R_6^0$$

Spherical Wrist

- for the spherical wrist

$$T_6^3 = T_4^3 T_5^4 T_6^5 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{if} \quad s_5 \neq 0$$

$$\theta_5^{\text{pos}} = \text{atan2}\left(\sqrt{1-r_{33}^2}, r_{33}\right)$$

$$\theta_5^{\text{neg}} = \text{atan2}\left(-\sqrt{1-r_{33}^2}, r_{33}\right)$$

Spherical Wrist

$$T_6^3 = T_4^3 T_5^4 T_6^5 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{for} \quad \theta_5^{\text{pos}}, \quad s_5 > 0$$

$$\theta_4 = \text{atan2}\left(r_{23}, r_{13}\right)$$

$$\theta_6 = \text{atan2}\left(r_{32}, -r_{31}\right)$$

Spherical Wrist

$$T_6^3 = T_4^3 T_5^4 T_6^5 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{for} \quad \theta_5^{\text{neg}}, \quad s_5 < 0$$

$$\theta_4 = \text{atan2}\left(-r_{23}, -r_{13}\right)$$

$$\theta_6 = \text{atan2}\left(-r_{32}, r_{31}\right)$$

Spherical Wrist

- if $\theta_5 = 0$

$$
{}^{3}_{6}T = {}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c_4c_6 - s_4s_6 & -c_4s_6 - s_4c_6 & 0 & 0 \\ s_4c_6 + c_4s_6 & -s_4s_6 + c_4c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Spherical Wrist

- continued from previous slide

$$= \begin{bmatrix} c_4 c_6 - s_4 s_6 & -c_4 s_6 - s_4 c_6 & 0 & 0 \\ s_4 c_6 + c_4 s_6 & -s_4 s_6 + c_4 c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{4+6} & -s_{4+6} & 0 & 0 \\ s_{4+6} & c_{4+6} & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

only the sum $\theta_4+\theta_6$ can be determined

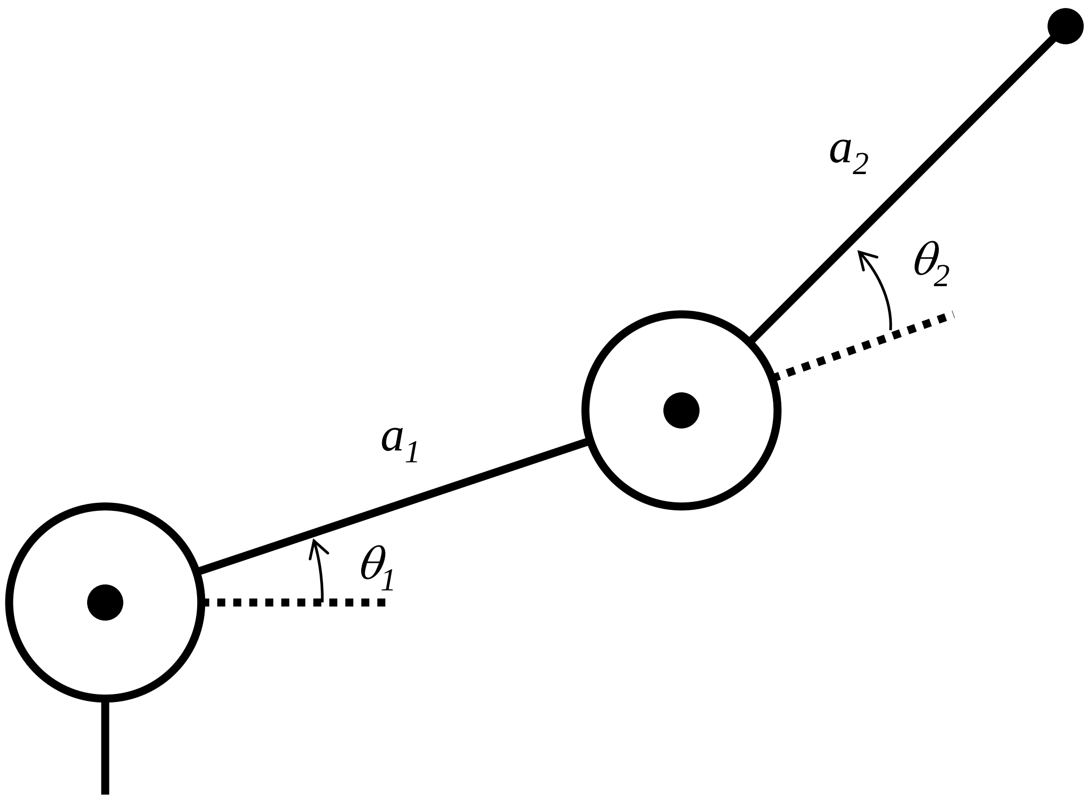
Using Inverse Kinematics in Path Generation

Path Generation

- a path is defined as a sequence of configurations a robot makes to go from one place to another
- a trajectory is a path where the velocity and acceleration along the path also matter

Path Generation

- consider the 2-link RR robot

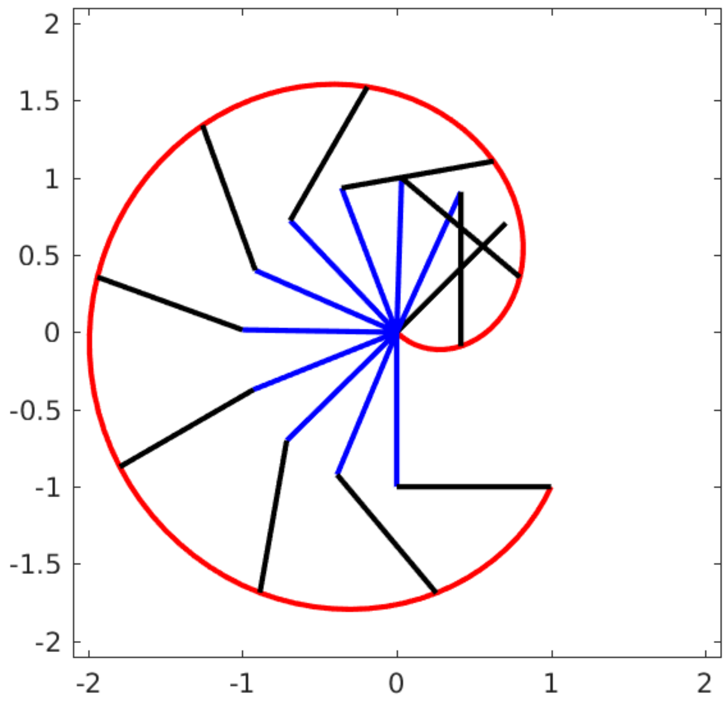


Path Generation

- suppose that the
  - initial configuration $\theta_1 = 45$ and $\theta_2 = -180$
  - final configuration $\theta_1 = 270$ and $\theta_2 = 90$

Joint-Space Path

- a joint-space path is computed considering the joint variables



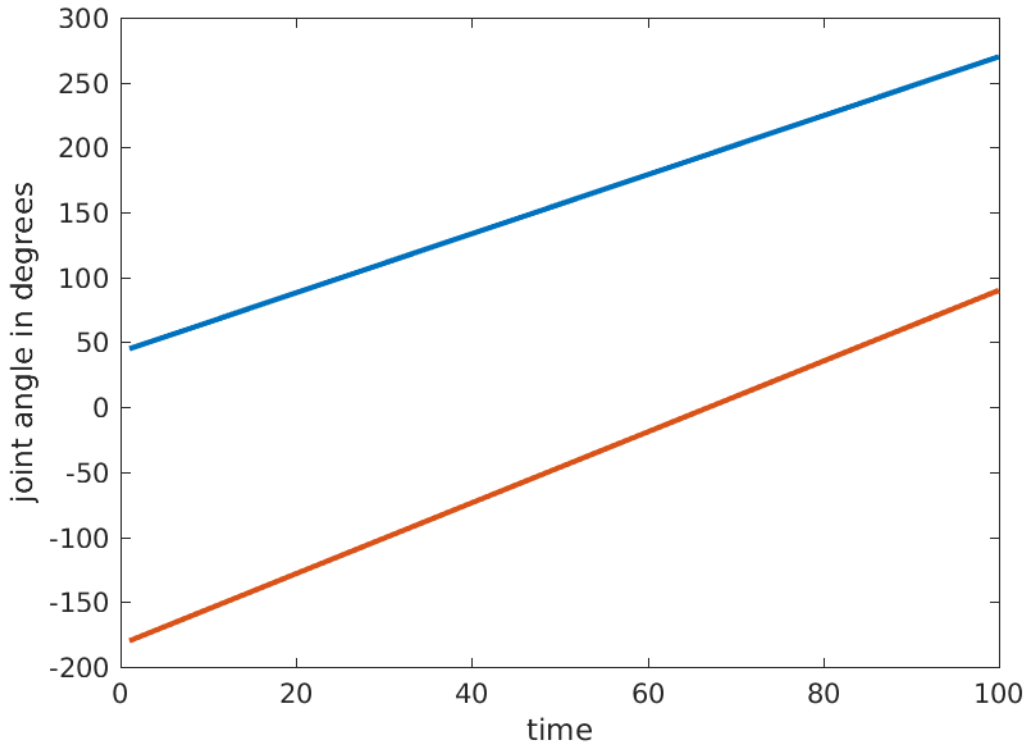
link 1

link 2

end effector path

Joint-Space Path Joint Angles

- linear joint-space path



Joint-Space Path

- given the current end-effector pose

$$ {}^{0}T $$

  and the desired final end-effector pose

$$ {}^{f}T $$

  find a sequence of joint angles that generates the path between the two poses
- idea
  - solve for the inverse kinematics for the current and final pose to get the joint angles for the current and final pose
  - interpolate the joint angles

Joint-Space Path

$$^{0}T \Rightarrow \text{inverse kinematics} \Rightarrow {}^{0}Q = {}^{0}\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}$$

$$^{f}T \Rightarrow \text{inverse kinematics} \Rightarrow {}^{f}Q = {}^{f}\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}$$

Joint-Space Path

find ${}^{0}Q$ from ${}^{0}T$ (inverse kinematics)

find ${}^{f}Q$ from ${}^{f}T$ (inverse kinematics)

$\Delta t = 1 / m$ where m is the number of steps in the path

$\Delta Q = {}^{f}Q - {}^{0}Q$

for $j = 1$ to m

  $t_j = j \Delta t$

  ${}^{j}Q = {}^{0}Q + t_j \Delta Q$

  set joints to ${}^{j}Q$

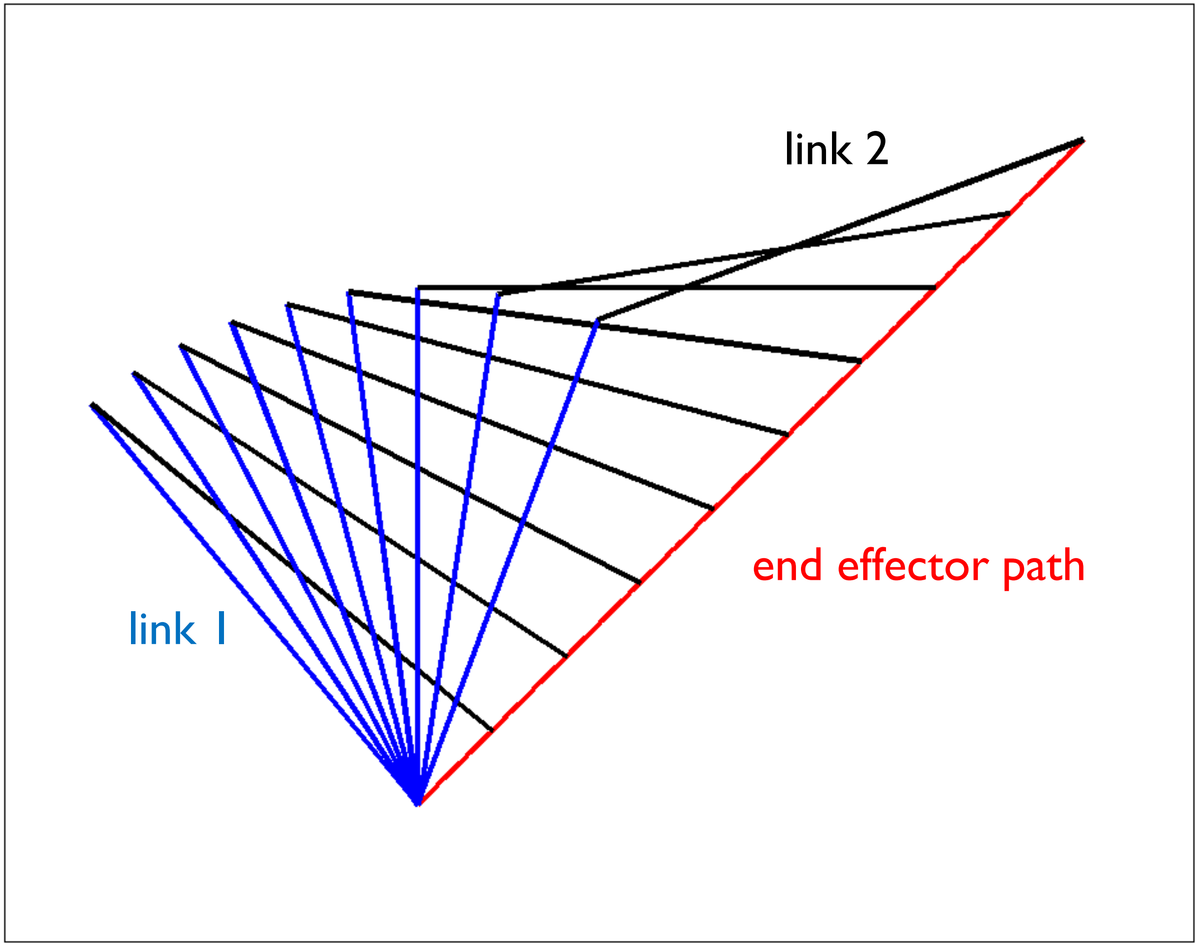
end

Joint-Space Path

- linearly interpolating the joint variables produces
  - a linear joint-space path
  - a non-linear Cartesian path
- depending on the kinematic structure the Cartesian path can be very complicated
  - some applications might benefit from a simple, or well defined, Cartesian path
    - e.g., move in a straight line, trace the shape of an object, move around objects in the workspace

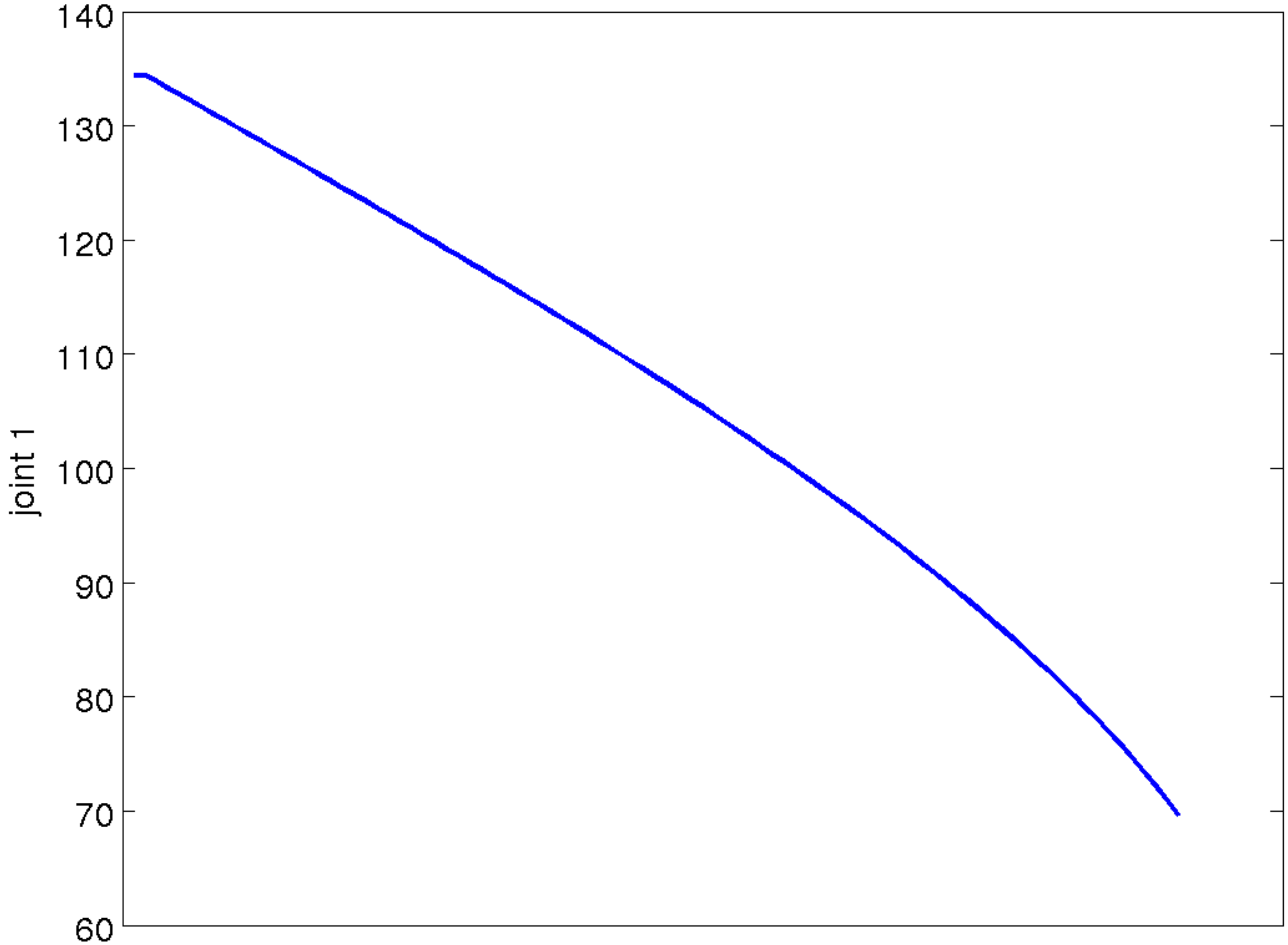
Cartesian-Space Path

- a Cartesian-space path considers the position of end-effector



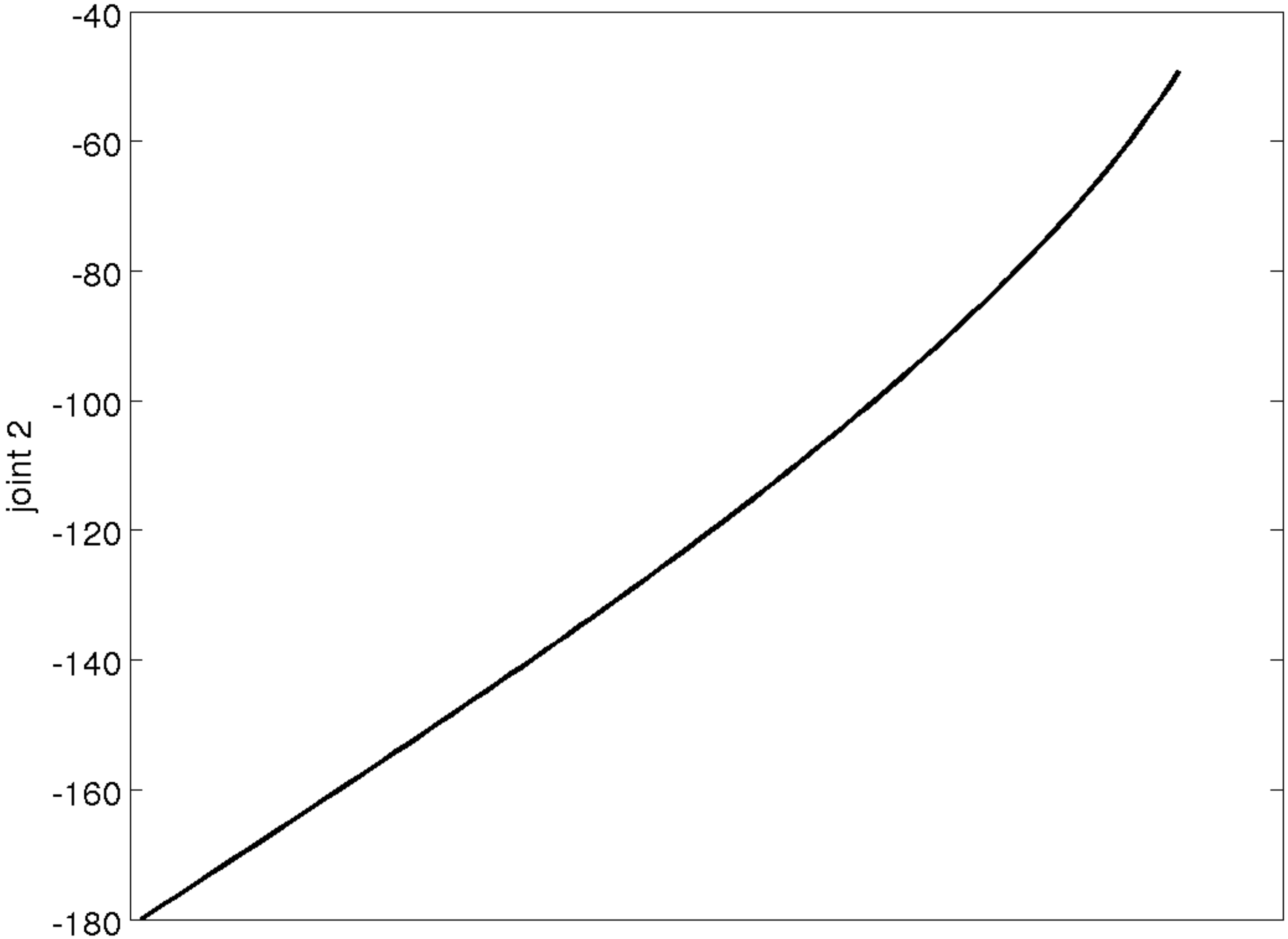
Cartesian-Space Path Joint Variable 1

- non-linear joint-space path



Cartesian-Space Path Joint Variable 2

- non-linear joint-space path



Cartesian-Space Straight Line Path

- given the current end-effector pose

$$^{0}p$$

  and the desired final end-effector position

$$^{f}p$$

  find a sequence of joint angles that generates the straight line path between the two points
- idea
  - interpolate the between the current and final positions
  - solve for the inverse kinematics at the interpolated positions to get the joint angles for the interpolated positions

Cartesian-Space Straight Line Path

$\Delta t = 1 / m$ where m is the number of steps in the path

$\Delta p = {}^f p - {}^0 p$

for $j = 1$ to m

  $t_j = j \, \Delta t$

  ${}^j p = {}^0 p + t_j \, \Delta p$

  find ${}^j Q$ from ${}^f p$ (inverse kinematics)

  set joints to ${}^j Q$

end

Cartesian-Space Path

- the previous straight line path algorithm only considered the position of the end effector
  - why?
- what problem do you need to solve if you need to consider the pose of the end effector?